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Observer-Based Controller Design for MIMO Two-Dimensional Systems

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Abstract

State feedback approach is a very popular design method in control engineering, however, the state variables are sometimes not accessible for direct measurement or the number of measuring devices is limited. Therefore, the observer-based feedback method will be extensively applied to the control system design because of its greater design flexibility. The purpose of this paper is to design a new observer-based controller so that the closed-loop characteristic polynomial of a multi-input multi-output (MIMO) two-dimensional (2-D) system is the same as the desired one. The system configuration described here is an observer-based feedback configuration, and the feedback compensator is of the form similar to the PID type for analog system. Furthermore, it is in expectation that these results can be used to design some practical system. Finally, an example is given to illustrate the feasibility of the developed approach.

Keywords: Observer-based feedback, PID controller, 2-D systems

1. Introduction

In control engineering, pole assignment approach is a very popular method for improving the behavior of a closed-loop system. It has drawn considerable attention because the design approach begins with a determination of the desired closed-loop poles based on transient-response requirements, such as, transient-response speed or damping ratio. Then, by choosing an appropriate feedback compensator, it is possible to force the system to have closed-loop poles at the desired locations, such that the system dynamics have the good performance. Therefore, as pointed out in the related literatures, the solution of pole assignment problem for the one-dimensional (1-D) system is very well known (Palm 2003, Ogata 1998, Phillips and Harbor 2000, Gopal 2003, Goodwin *et al*. 2001). Though the 2-D system design problem has been shown in several papers (Agathoklis and Foda 1988, Shimonishi *et al*. 1989, Liu 2013, Liu and Chung 2015), most of them make use of state feedback method. For example, Agathoklis and Foda (1988) did the characteristic polynomial assignment by the state feedback method. Shimonishi *et al*. (1989) also discussed the problem by using an idea that the eigenvalue assignment problem of 2-D systems is divided into two 1-D eigenvalue assignment problems. Recently, Liu (2013) studied the controller gain design problem of 2-D linear systems based on the existing 1-D analytical solution. More, Liu and Chung (2015) dealt with the state observer design problem for two-dimensional discrete systems by using Ackermann's formula. In the state feedback method, all the state variables are assumed to be measurable and be available for feedback, however, this assumption generally does not hold in practice, because the state variables are not accessible for direct measurement or the number of measuring devices is limited. Fortunately, Liu (2012) studied the 2-D controller design problem via observer-based feedback control. Based on the known system inputs and the outputs that can be measured, the closed-loop characteristic polynomial of a 2-D system has desired pre-assigned eigenvalues. However, the compensator developed by Liu (2012) just restricted to a constant form.

In this paper, in order to overcome the problems stated above, based on the results shown by Liu (2012), we will tackle the 2-D control design problem by means of the observer-based feedback method (Chen 1987), and it has been extensively applied to the control system design because of its greater design flexibility. Furthermore, the developed controllers will be extended from constant form to a form similar to the PID type for analog systems, which will result in a large number of free parameters, and make the design system specifications more flexible. It should be pointed out that the design procedures for MIMO PID controllers are very difficult than those in Liu (2012), and the overall closed loop system developed in this paper is a generalized form of the system which tackled the same issue using the state or output feedback method.

2. Description of the system and problem

Consider the 2-D linear system described by the following state space model (Roessor 1975)

$$\begin{bmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(i, j) \tag{1}$$

$$y(i, j) = [C_1 \quad C_2] \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix} \tag{2}$$

or more compactly

$$x'(i, j) = Ax(i, j) + Bu(i, j) \tag{3}$$

$$y(i, j) = Cx(i, j) \tag{4}$$

where $x^h(i, j)$ is the $n_1 \times 1$ horizontal state vector, $x^v(i, j)$ is the $n_2 \times 1$vertical state vector, $x(i, j)$ is the $(n_1 + n_2) \times 1$ local state vector, $u(i, j)$ is the $t \times 1$ control vector, $y(i, j)$ is the $s \times 1$ output vector. A, B and C are real matrices of appropriate dimensions, $x(i, j)$ and $x'(i, j)$ are denoted as follows.

$$x(i, j) = \begin{bmatrix} x^h(i, j) \\ x^v(i, j) \end{bmatrix}, \quad x'(i, j) = \begin{bmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{bmatrix}$$

A 2-D linear shift-invariant plant input-output feedback system is shown in figure 1, which is composed of a plant with transfer function $P(z_1, z_2)$, a feed-forward controller $E(z_1, z_2)$ and a feedback controller $F(z_1, z_2)$. And $V(z_1, z_2)$ is a (z_1, z_2)-transform of a new t-dimensional input vector $v(i, j)$.

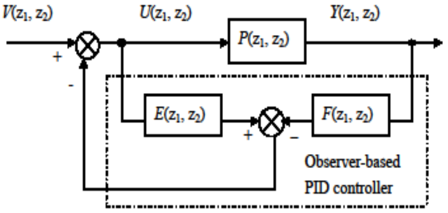

Figure 1. A Plant input-output feedback system

From Figure 1, one can derive the control law as follows:

$$U(z_1, z_2) = V(z_1, z_2) - E(z_1, z_2)U(z_1, z_2) + F(z_1, z_2)Y(z_1, z_2) \tag{5}$$

where $U(z_1, z_2)$ and $Y(z_1, z_2)$ are the (z_1, z_2)-transform of $u(i, j)$ and $y(i, j)$, respectively. $E(z_1, z_2)$ and $F(z_1, z_2)$ are 2-D polynomial matrices with dimension $t \times t$ and $t \times s$, respectively. The transfer function of the open-loop system (1) and (2) is given as follows:

$$P(z_1, z_2) = \frac{CH(z_1, z_2)}{d_o(z_1, z_2)} \tag{6}$$

where $d_o(z_1, z_2)$ is an open-loop characteristic polynomial and $H(z_1,z_2)$ is a polynomial matrix with dimension $(n_1 + n_2) \times t$, which can be expressed as follows.

$$H(\mathrm{z}_1, \mathrm{z}_2) = \{\mathrm{Adj}[\mathrm{z}I - A]\}B = \sum_{i=0}^{n1} \sum_{j=0}^{n2} h_{ij}\mathrm{z}_1^i\mathrm{z}_2^j \quad \text{with } h_{n1n2} = 0 \tag{7}$$

and

$$d_o(\mathrm{z}_1, \mathrm{z}_2) = \det[\mathrm{z}I - A] = \sum_{i=0}^{n1} \sum_{j=0}^{n2} d_{o(ij)}\, \mathrm{z}_1^i\mathrm{z}_2^j \quad \text{with } d_{o(n1n2)} \neq 0 \tag{8}$$

here

$$\mathrm{z}I = \begin{bmatrix} \mathrm{z}_1 I_{n1} & 0 \\ 0 & \mathrm{z}_2 I_{n2} \end{bmatrix} \tag{9}$$

Then the closed-loop characteristic polynomial $d_c(\mathrm{z}_1, \mathrm{z}_2)$ will be

$$d_c(\mathrm{z}_1, \mathrm{z}_2) = \det[\mathrm{z}I - A - B(I_t + E(\mathrm{z}_1, \mathrm{z}_2))^{-1}F(\mathrm{z}_1, \mathrm{z}_2)C]\ \det[I_t + E(\mathrm{z}_1, \mathrm{z}_2)]$$

$$= \sum_{i=0}^{n1} \sum_{j=0}^{n2} d_{c(ij)}\, \mathrm{z}_1^i\mathrm{z}_2^j \quad \text{with } d_{c(n1n2)} \neq 0 \tag{10}$$

where I_t is the identity matrix with dimension $t \times t$.

Let $d_d(\mathrm{z}_1, \mathrm{z}_2)$ be the desired 2-D characteristic polynomial of the closed-loop system. At this point, we can state the problem as follows: Given A, B, C and $d_d(\mathrm{z}_1, \mathrm{z}_2)$, find the controller matrices $E(\mathrm{z}_1, \mathrm{z}_2)$ and $F(\mathrm{z}_1, \mathrm{z}_2)$, such that

$$d_c(\mathrm{z}_1, \mathrm{z}_2) = d_d(\mathrm{z}_1, \mathrm{z}_2) \tag{11}$$

3. Concise results

In this section, based on the results derived in Liu (2012), the developed controllers will be extended from constant form to PID form, which will result in a large number of free parameters, and make the design system specifications more flexible. Furthermore, in next section, a difficult example will be given to illustrate the feasibility of the developed approach.

Here, we tackle $E(\mathrm{z}_1, \mathrm{z}_2)$ and $F(\mathrm{z}_1, \mathrm{z}_2)$ with the following PID structures

$$E(\mathrm{z}_1, \mathrm{z}_2) = E \tag{12}$$

$$F(\mathrm{z}_1, \mathrm{z}_2) = \frac{F_1}{\mathrm{z}_1} + \frac{F_2}{\mathrm{z}_2} + F_3 + \mathrm{z}_1F_4 + \mathrm{z}_2F_5 \tag{13}$$

here, the matrices F_1, F_2, F_3, F_4, F_5 are with dimensions $t \times s$. Substituting (12) and (13) into (10), and $\det[I_t + E] = 1/k$ is a real value, more let $(I_t + E)^{-1} = K$ and $KF(\mathrm{z}_1, \mathrm{z}_2) = L(\mathrm{z}_1, \mathrm{z}_2)$. Then (10) will be equal to the following equation

$$kd_c(\mathrm{z}_1, \mathrm{z}_2) = \det[\mathrm{z}I - A - BL(\mathrm{z}_1, \mathrm{z}_2)C] \tag{14}$$

Let $L(\mathrm{z}_1, \mathrm{z}_2)$ has the following dyadic form:

$$L(\mathrm{z}_1, \mathrm{z}_2) = gw^{\mathrm{T}}(\mathrm{z}_1, \mathrm{z}_2) = g\frac{w_1^{\mathrm{T}}}{\mathrm{z}_1} + g\frac{w_2^{\mathrm{T}}}{\mathrm{z}_2} + gw^{\mathrm{T}}{}_3 + \mathrm{z}_1 gw^{\mathrm{T}}{}_4 + \mathrm{z}_2 gw^{\mathrm{T}}{}_5 \tag{15}$$

where g is a given $t \times 1$ vector and $w_m (m = 1, 2, \ldots, 5)$ are $s \times 1$ vectors to be determined such that the characteristic polynomial of the closed system is equal to the desired one.

Now, substituting (15) into (14), after a simple process, the following equation can be derived.

$$\mathrm{z}_2\underline{H}^{\mathrm{T}}(\mathrm{z}_1, \mathrm{z}_2)C^{\mathrm{T}}w_1 + \mathrm{z}_1\underline{H}^{\mathrm{T}}(\mathrm{z}_1, \mathrm{z}_2)C^{\mathrm{T}}w_2 + \mathrm{z}_1\mathrm{z}_2\underline{H}^{\mathrm{T}}(\mathrm{z}_1, \mathrm{z}_2)C^{\mathrm{T}}w_3 + \mathrm{z}_1{}^2\mathrm{z}_2\, \underline{H}^{\mathrm{T}}(\mathrm{z}_1, \mathrm{z}_2)C^{\mathrm{T}}w_4 + \mathrm{z}_1\mathrm{z}_2{}^2\underline{H}^{\mathrm{T}}(\mathrm{z}_1, \mathrm{z}_2)C^{\mathrm{T}}w^5 = \mathrm{z}_1\mathrm{z}_2\, \bar{d}\, (\mathrm{z}_1, \mathrm{z}_2) \tag{16}$$

In order to solve (16), $\underline{H}^{\mathrm{T}}(\mathrm{z}_1, \mathrm{z}_2)C^{\mathrm{T}}$ may be written as

$$\underline{H}^{\mathrm{T}}(\mathrm{z}_1, \mathrm{z}_2)C^{\mathrm{T}} = \sum_{i=0}^{n1} \sum_{j=0}^{n2} \underline{h}_{ij}C^{\mathrm{T}}\mathrm{z}_1^i\mathrm{z}_2^j = W_{n1}\underline{H}^{\mathrm{T}}\, X_{n2} \tag{17}$$

where $\underline{H}^{\mathrm{T}}(\mathrm{z}_1, \mathrm{z}_2)C^{\mathrm{T}}$ is $1\times s$ matrix, and

$$W_{n1} = \begin{bmatrix} 1 \\ \mathrm{z}_1 \\ \cdots \\ \mathrm{z}_1^{n1} \end{bmatrix}^{\mathrm{T}}, \quad X_{n2} = \begin{bmatrix} I_{\mathrm{s}} \\ \mathrm{z}_2\, I_{\mathrm{s}} \\ \cdots \\ \mathrm{z}_2^{n2}\, I_{\mathrm{s}} \end{bmatrix}, \quad \underline{H}^{\mathrm{T}} = \begin{bmatrix} \underline{h}_{00}^{\mathrm{T}}C^{\mathrm{T}} & \underline{h}_{01}^{\mathrm{T}}C^{\mathrm{T}} & \cdots & \underline{h}_{0n2}^{\mathrm{T}}C^{\mathrm{T}} \\ \underline{h}_{10}^{\mathrm{T}}C^{\mathrm{T}} & \cdots & & \cdots \\ \cdots & & & \\ \underline{h}_{n10}^{\mathrm{T}}C^{\mathrm{T}} & \cdots & & \underline{h}_{n1n2}^{\mathrm{T}}C^{\mathrm{T}} \end{bmatrix} \tag{18}$$

and $\underline{H}^{\mathrm{T}}$ is a $[(n_1 + 1) \times (n_2 + 1)s]$ matrix.

Let $G_{ab}(\mathrm{z}_1, \mathrm{z}_2) = \mathrm{z}_1{}^a\mathrm{z}_2{}^b\underline{H}^{\mathrm{T}}(\mathrm{z}_1, \mathrm{z}_2)C^{\mathrm{T}} = W_{n1+a}G_{ab}X_{n2+b}$

$$= \sum_{i=0}^{n1+a} \sum_{j=0}^{n2+b} (G_{ab})_{ij}\mathrm{z}_1^i\mathrm{z}_2^j \tag{19}$$

where

$$W_{n1+a} = [1\ \mathrm{z}_1\ \ \ldots\ \mathrm{z}_1{}^{n1+a}], \quad X_{n2+b} = \begin{bmatrix} I_{\mathrm{s}} \\ \mathrm{z}_2\, I_{\mathrm{s}} \\ \cdots \\ \mathrm{z}_2^{n2+b}\, I_{\mathrm{s}} \end{bmatrix},$$

$$G_{ab} = \begin{bmatrix} (G_{ab})_{00} & (G_{ab})_{01} & \cdots & (G_{ab})_{0,n2+b} \\ (G_{ab})_{10} & \cdots & & \cdots \\ \cdots & & & \\ (G_{ab})_{n1+a,0} & \cdots & & (G_{ab})_{n1+a,n2+b} \end{bmatrix}$$

$$=\begin{array}{cc} \begin{array}{cc}\overbrace{\hphantom{0_{1\times s}\ \cdots\ 0_{1\times s}}}^{b\ \text{blocks}} & \overbrace{\hphantom{0_{1\times s}\ \cdots\ 0_{1\times s}}}^{n_2+1\ \text{blocks}}\end{array} & \\ \left[\begin{array}{cccccc} 0_{1\times s} & \cdots & 0_{1\times s} & 0_{1\times s} & \cdots & 0_{1\times s}\\ \cdots & & \cdots & \cdots & & \cdots \\ 0_{1\times s} & \cdots & 0_{1\times s} & 0_{1\times s} & \cdots & 0_{1\times s}\\ 0_{1\times s} & \cdots & 0_{1\times s} & & & \\ \cdots & & \cdots & & H^{\mathrm{T}} & \\ 0_{1\times s} & \cdots & 0_{1\times s} & & & \end{array}\right]. & \begin{array}{l}\left.\vphantom{\begin{array}{c}0\\0\\0\end{array}}\right\}a\ \ \text{blocks}\\ \left.\vphantom{\begin{array}{c}0\\0\\0\end{array}}\right\}n_1+1\ \ \text{blocks}\end{array}\end{array} \tag{20}$$

and G_{ab} is a $[(n_1 + a + 1) \times (n_2 + b + 1)s]$ matrix.

Now, Substituting (17)-(20) into (16), and let $\sigma_1 = n_1 + 2$, $\sigma_2 = n_2 + 2$, we have

$$[\sum_{i=0}^{\sigma 1}\sum_{j=0}^{\sigma 2}(G_{01})_{ij}z_1^i z_2^j]w_1+[\sum_{i=0}^{\sigma 1}\sum_{j=0}^{\sigma 2}(G_{10})_{ij}z_1^i z_2^j]w_2+$$

$$[\sum_{i=0}^{\sigma 1}\sum_{j=0}^{\sigma 2}(G_{11})_{ij}z_1^i z_2^j]w_3+[\sum_{i=0}^{\sigma 1}\sum_{j=0}^{\sigma 2}(G_{21})_{ij}z_1^i z_2^j]w_4+$$

$$[\sum_{i=0}^{\sigma 1}\sum_{j=0}^{\sigma 2}(G_{12})_{ij}z_1^i z_2^j]w_5=\sum_{i=0}^{\sigma 1}\sum_{j=0}^{\sigma 2}\bar{d}_{ij}z_1^i z_2^j \tag{21}$$

Equating coefficients of the term $z_1^i z_2^j$ on both sides of (21), we obtain

$$(G_{01})_{ij}w_1+(G_{10})_{ij}w_2+(G_{11})_{ij}w_3+(G_{21})_{ij}w_4+(G_{12})_{ij}w_5=\bar{d}_{ij} \tag{22}$$

Thus, (21) may lead to a linear system of algebraic equations of the following form

$$Gw=\bar{d} \tag{23}$$

where

$$G=\begin{bmatrix}(G_{01})_{00} & (G_{10})_{00} & (G_{11})_{00} & (G_{21})_{00} & (G_{12})_{00}\\ (G_{01})_{01} & (G_{10})_{01} & (G_{11})_{01} & (G_{21})_{01} & (G_{12})_{01}\\ (G_{01})_{02} & (G_{10})_{02} & (G_{11})_{02} & (G_{21})_{02} & (G_{12})_{02}\\ (G_{01})_{03} & (G_{10})_{03} & (G_{11})_{03} & (G_{21})_{03} & (G_{12})_{03}\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ (G_{01})_{\sigma 1\sigma 2} & (G_{10})_{\sigma 1\sigma 2} & (G_{11})_{\sigma 1\sigma 2} & (G_{21})_{\sigma 1\sigma 2} & (G_{12})_{\sigma 1\sigma 2}\end{bmatrix},$$

$$\bar{d}=\begin{bmatrix}\bar{d}_{00}\\ \bar{d}_{01}\\ \bar{d}_{02}\\ \bar{d}_{03}\\ \cdots\\ \bar{d}_{\sigma 1\sigma 2}\end{bmatrix},\quad w=\begin{bmatrix}w_1\\ w_2\\ w_3\\ w_4\\ w_5\end{bmatrix} \tag{24}$$

One can observe that (23) has $N = (n_1 + 3)(n_2 + 3) – 1$ equations with $5s$ unknowns. Then the 2-D characteristic polynomial assignment problem has a solution if and only if there exists a solution w to (23).

Now, we are in a position to establish the following Theorem.

Theorem: Given the systems (1), (2) and the desired closed-loop characteristic polynomial $d_d(z_1, z_2)$, applying the control law (5), we can find the controller matrices $E(z_1, z_2)$ (12) and $F(z_1, z_2)$ (13), such that the characteristic polynomial $d_c(z_1, z_2)$ of the closed-loop system will be desired, i.e. $d_c(z_1, z_2) = d_d(z_1, z_2)$.

Proof: Refer to Liu (2012).

4. Illustrative example

Consider the multi-input multi-output 2-D system described by (1) and (2).

Given

$$A=\begin{bmatrix}0 & -1\\ 2 & 1\end{bmatrix},\quad B=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix},\quad C=\begin{bmatrix}1 & 1\\ -1 & 0\end{bmatrix} \tag{25}$$

and the desired closed-loop characteristic polynomial $d_d(z_1, z_2)$ be as follows

$$d_d(z_1, z_2) = z_1z_2 – 0.8\, z_1 – 0.5z_2 + 0.4 \tag{26}$$

We have $n_1 = n_2 = 1$, $t = 2$, $s = 2$

and $d_o(z_1, z_2) = \det[zI – A] = z_1z_2 – z_1 + 2$

and

$$H(z_1, z_2) = \{\mathrm{Adj}[zI – A]\}= \begin{bmatrix}-1 & z_2-1\\ z_1 & 2\end{bmatrix},$$

Choosing $g=\begin{bmatrix}1\\ -1\end{bmatrix}$, then $\underline{H}(z_1, z_2) = H(z_1, z_2)g=\begin{bmatrix}-z_2\\ z_1-2\end{bmatrix}$,

So,

$$\underline{h}_{00}^{\mathrm{T}}=[0\ \ -2],\ \underline{h}_{01}^{\mathrm{T}}=[-1\ \ 0],\ \underline{h}_{10}^{\mathrm{T}}=[0\ \ 1],\ \underline{h}_{11}^{\mathrm{T}}=[0\ \ 0],$$

Hence

$$\underline{H}^{\mathrm{T}}=\begin{bmatrix}-2 & 0 & -1 & 1\\ 1 & 0 & 0 & 0\end{bmatrix}$$

therefore,

$$G_{01}=\begin{bmatrix}(0\ \ 0) & (-2\ \ 0) & (-1\ \ 1)\\ (0\ \ 0) & (1\ \ 0) & (0\ \ 0)\end{bmatrix},\quad G_{10}=\begin{bmatrix}(0\ \ 0) & (0\ \ 0)\\ (-2\ \ 0) & (-1\ \ 1)\\ (1\ \ 0) & (0\ \ 0)\end{bmatrix},$$

$$G_{21}=\begin{bmatrix}(0\ \ 0) & (0\ \ 0) & (0\ \ 0)\\ (0\ \ 0) & (0\ \ 0) & (0\ \ 0)\\ (0\ \ 0) & (-2\ \ 0) & (-1\ \ 1)\\ (0\ \ 0) & (1\ \ 0) & (0\ \ 0)\end{bmatrix},\quad G_{11}=\begin{bmatrix}(0\ \ 0) & (0\ \ 0) & (0\ \ 0)\\ (0\ \ 0) & (-2\ \ 0) & (-1\ \ 1)\\ (0\ \ 0) & (1\ \ 0) & (0\ \ 0)\end{bmatrix},$$

$$G_{12}=\begin{bmatrix}(0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0)\\ (0\quad 0) & (0\quad 0) & (-2\quad 0) & (-1\quad 1)\\ (0\quad 0) & (0\quad 0) & (1\quad 0) & (0\quad 0)\end{bmatrix},$$

By (15), we choose $K=\begin{bmatrix}0 & 1\\ -1 & 1\end{bmatrix}$, and derive $E=\begin{bmatrix}0 & -1\\ 1 & -1\end{bmatrix}$.

So, $\det[I_t + E] = 1$, then $k = 1$.

Now, one can calculate $\bar{d}\,(z_1, z_2)$ as follows

$$\bar{d}\,(z_1, z_2) = d_O(z_1, z_2) - kd_d(z_1, z_2) = -0.2z_1 + 0.5z_2 + 1.6$$

The matrices G and $\bar{d}_1$ are as follows

$$G=\begin{bmatrix}(0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0)\\ (-2\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0)\\ (-1\quad 1) & (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0)\\ (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0)\\ (0\quad 0) & (-2\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0)\\ (1\quad 0) & (-1\quad 0) & (-2\quad 0) & (0\quad 0) & (0\quad 0)\\ (0\quad 0) & (0\quad 0) & (-1\quad 1) & (0\quad 0) & (-2\quad 0)\\ (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0) & (-1\quad 1)\\ (0\quad 0) & (1\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0)\\ (0\quad 0) & (0\quad 0) & (1\quad 0) & (-2\quad 0) & (0\quad 0)\\ (0\quad 0) & (0\quad 0) & (0\quad 0) & (-1\quad 1) & (1\quad 0)\\ (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0)\\ (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0)\\ (0\quad 0) & (0\quad 0) & (0\quad 0) & (1\quad 0) & (0\quad 0)\\ (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0)\\ (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0) & (0\quad 0)\end{bmatrix},$$

$$\bar{d} = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1.6 & 0.5 & 0 & 0 & -0.2 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T,$$

$$w^T = [w_1{}^T\, w_2{}^T\, w_3{}^T\, w_4{}^T\, w_5{}^T\,]$$

Now, solving the system (33), we get

$$w^T = [w_1{}^T\, w_2{}^T\, w_3{}^T\, w_4{}^T\, w_5{}^T\,] = [(0\quad 0)(0\quad 1.2)(\,-0.2\quad 2.3)(0\quad -1)(1\quad 1)]$$

By (18) and (16), we derive

$$F(z_1, z_2)=\begin{bmatrix}2z_2-0.4 & 2(z_2-z_1+\dfrac{1.2}{z_2}+2.3)\\ z_2-0.2 & -(z_1-z_2-\dfrac{1.2}{z_2}-2.3)\end{bmatrix}$$

In what follows, we will check the results.
Substituting E and $F(z_1, z_2)$ into (10), we get

$$\begin{aligned}d_c(z_1, z_2) &= \det[zI - A - B(I + E(z_1, z_2))^{-1}F(z_1, z_2)C]\,\det[I_t + E(z_1, z_2)]\\ &= z_1z_2 - 0.8z_1 - 0.5z_2 + 0.4\end{aligned}$$

It is obvious that the closed-loop characteristic polynomial is equal to the desired polynomial $d_d(z_1, z_2)$. Thus, E and $F(z_1, z_2)$ given above are the solutions.

5. Conclusions

In this paper, we considered the characteristic polynomial assignment problem for 2-D system expressed by Roesser's state-space model. In spite of these problems had been discussed in several papers, for the reasons of the techniques, approaches, mathematical means, model which described the systems and the system structure used to solve these problems are different, we devoted ourself to studying it deeply. Different configuration of the system plant input-output feedback system is adapted in this paper, and we treat the 2-D characteristic polynomial assignment problem by means of the observer-based feedback. It should be pointed out that the overall closed loop system developed in this paper is a generalized form of the system which tackled the same issue using the state or output feedback method. And the developed controllers have been extended from the constant form to a form similar to the PID type of analog systems, which will result in a large number of free parameters. Finally, we give an example to illustrate the feasibility of this approach.

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二維多輸入多輸出控制系統的觀測器式控制器設計

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摘　要

在控制系統領域中系統狀態回授控制方法是非常受歡迎且有用的方法，但系統狀態有時是不可以被量測的，很多的控制系統設計法則就無法被應用了。此時，因為具有更大的設計彈性，觀測器式回授法將可以被應用來解決控制系統的設計問題。本論文的目的在於設計一種新的觀測器式回授控制器，使得多輸入多輸出二維控制線性系統的閉迴路特性方程式與設計者想要的性能一致。在此，我們運用的系統架構是所謂的觀測器式回授架構，而回授控制器的型態是類似類比系統的 PID 控制器的型態。更進一步，我們期望這些結果可以應用到與實務上相關的控制系統上。最後，我們將舉例以說明本方法之效用。

關鍵詞：觀測器式回授、PID 控制器、二維控制系統